UCN and Exotic Interactions

Vladimir Gudkov
University of South Carolina

International Workshop on UCN and Fundamental Neutron Physics

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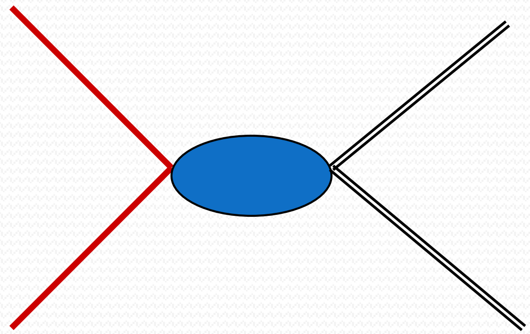
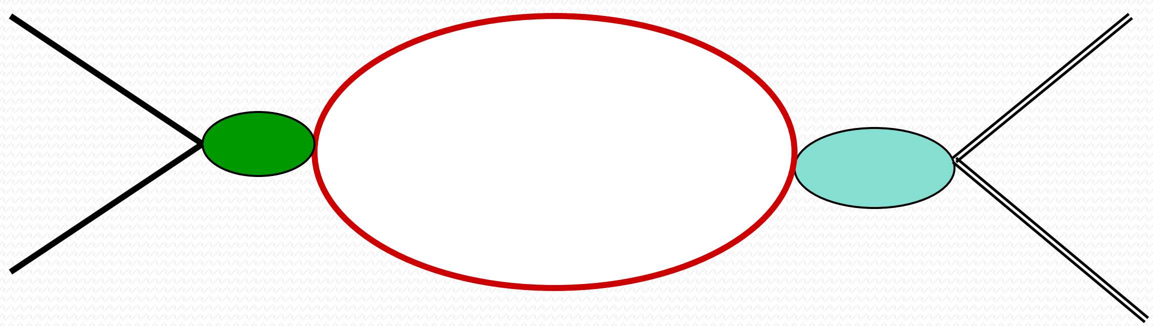
High Energy Physics with Low Energy Neutrons

HEP:



$E \rightarrow \infty$

FNP:



$\Delta E/E \rightarrow 0$

Some characteristic scales (UCN)

$$E_n = 100 neV \quad \Rightarrow \quad \lambda = 90.4 nm$$

$$E_n = 1 neV \quad \Rightarrow \quad \lambda = 904 nm$$

"Dark energy" density is $\sim(10^{-3}\, eV)^4$

Abnormal Gravity: sensitivity of neutrons for test of gravity at corresponding distances ($< 0.1\, mm$)

Extra dimensions and Gravity

$$R \sim M^{-1}\left(\frac{M_{Pl}}{M_*}\right)^{2/d} \sim 10^{32/d-17}\,cm$$

$$V_G(r) = -G\frac{mM}{r}\left(1+\alpha_G e^{-r/\lambda}\right)$$

$$V_G(r) = -G\frac{mM}{r}\left(1+\alpha_G\left(\frac{\lambda}{r}\right)^n\right)$$

Interferometry

$$\Delta\Phi = \Delta\Phi_{nuclear} + \Delta\Phi_{ne} + \Delta\Phi_{grav}$$

- Very small intensity of a signal
- Coherence
- Resonance (energy scale)

Diffraction

$$b_{coh} = b_N + Z\left[1 - f(q)\right]b_{ne} + f_G(q)b_G$$

- For q=0: $b_{coh} = b_N + b_G$
- Bragg reflection

$$F_{H_1} = \sqrt{32}(b_N + Z[1 - f(H_1)]b_{ne} + f_G(H_1)b_G)$$

$$F_{H_3} = \sqrt{32}(b_N + Z[1 - f(H_3)]b_{ne} + f_G(H_3)b_G)$$

$$b_G = -\frac{2m_n^2 MG\alpha_G\lambda^2}{\hbar^2}$$

G. L. Greene and V. Gudkov, Phys. Rev. C 75, 015501 (2007).

$$f_G(q) = \frac{1}{1+(q\lambda)^2}$$

$$f(H_1) = 0.7526 \qquad f(H_3) = 0.4600$$

- For $q^{-1} \sim 1\overset{\circ}{A}$

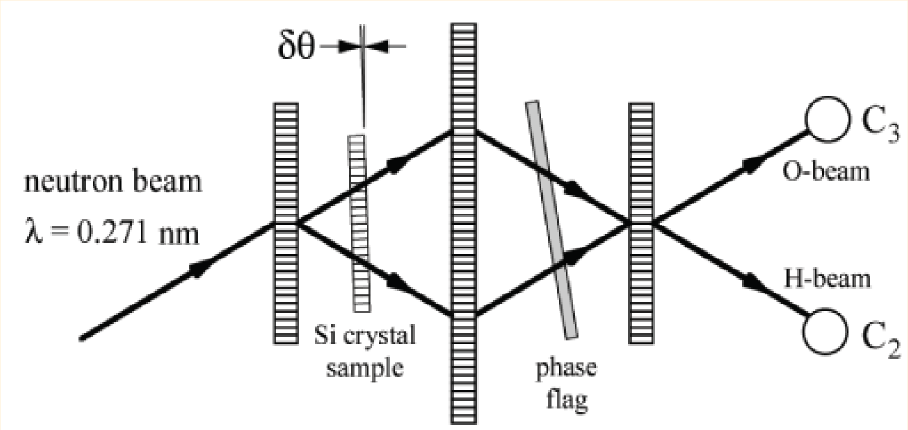
$$f_G(q)b_G = -\frac{2m_n^2 MG\alpha_G\lambda^2}{\hbar^2}$$

- For $\lambda \gg 1\overset{\circ}{A}$

$$f_G(q)b_G \simeq -\frac{2m_n^2 MG\alpha_G}{\hbar^2}\frac{1}{q^2}$$

$$b_G \simeq -1.6\times 10^{-6}(\alpha_G\lambda^2) \qquad \alpha_G\lambda^2 \leq 25.6m^2$$

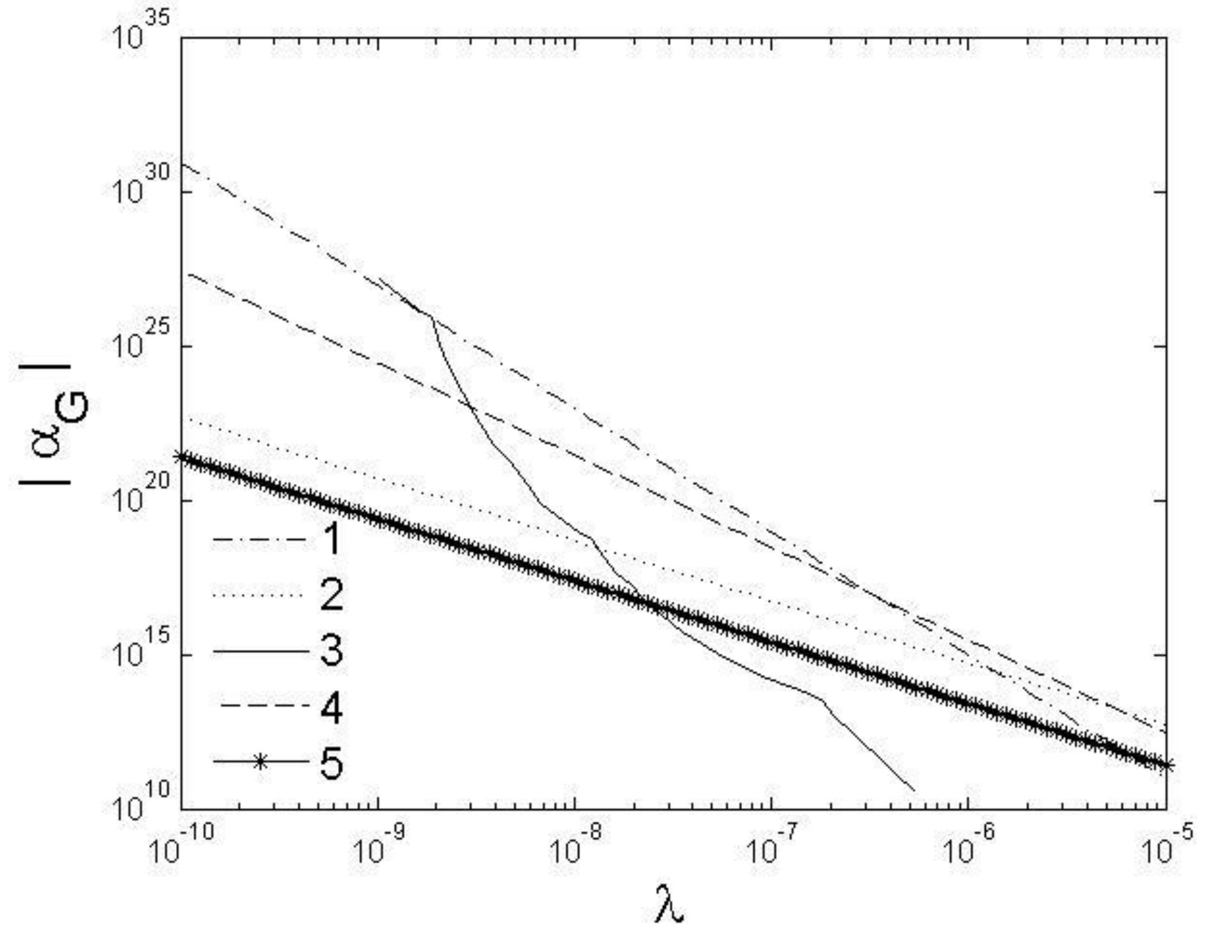
Neutron Interferometer Experiment



off Bragg: $b_{\text{coh}} = b_N + Z[1 - f(0)]b_{ne} = b_N$

near Bragg: $b_{\text{coh}} = b_N + Z[1 - f(\vec{H}_{111})]b_{ne}$

Courtesy of Fred Wietfeldt



(1) from Nesvizhevsky and Protasov;
(2) from Zimmer and Kaiser;
(3) from the review of Adelberger;
(4) from the “two-plate” method;
(5) from the diffraction method.

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Translation Method

$$V_{eff} = 2\pi G \alpha_G m_n \rho \lambda^2 e^{-x/\lambda}$$ outside the material;

$$V_{eff} = 2\pi G \alpha_G m_n \rho \lambda^2 (2 - e^{-x/\lambda})$$ inside the material.

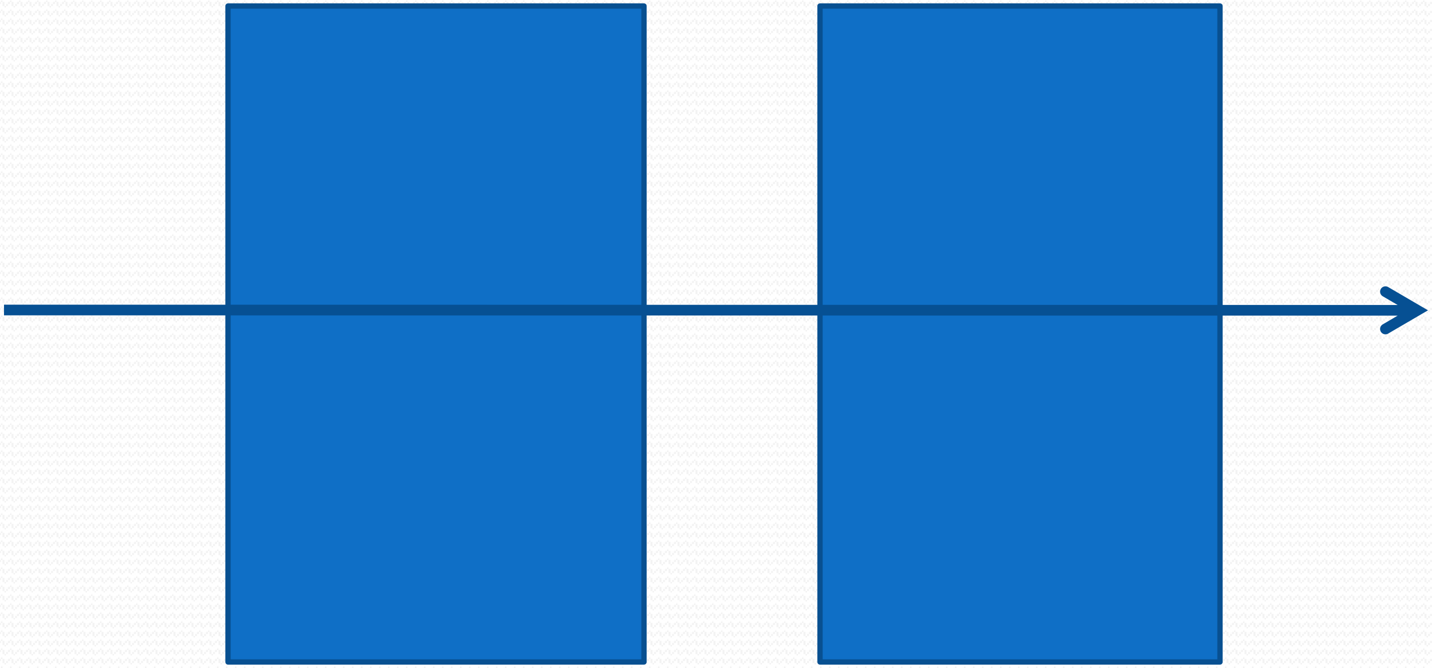
$$\Delta\Phi = \frac{4\pi G \alpha_G m_n \rho \lambda^3}{k_0} \left(\frac{2m_n}{\hbar^2} \right) \left(1 - e^{-L/\lambda}\right)$$

$$k_0 = \sqrt{\frac{2m_n}{\hbar^2} E_n}$$

Two plates + Gravity

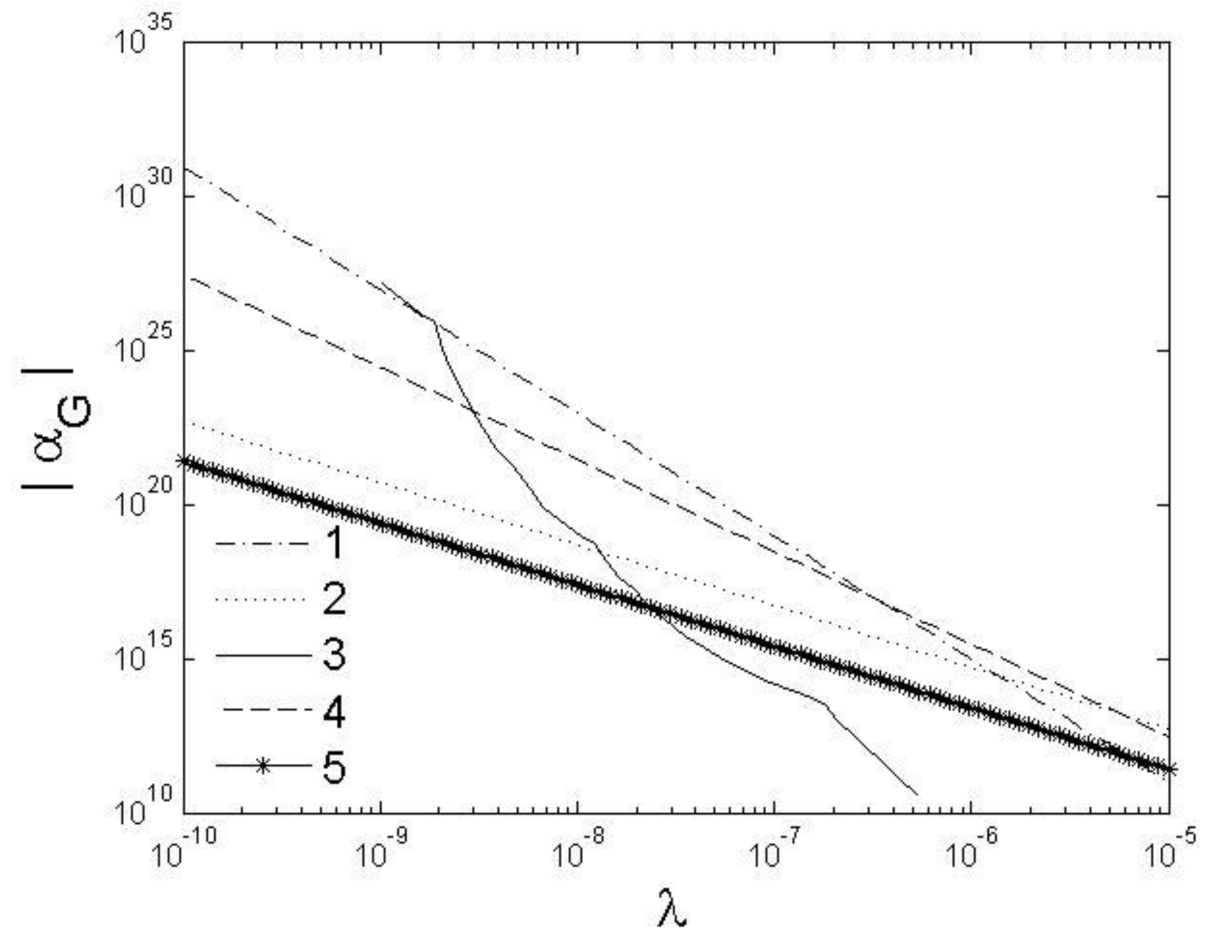
$$k^2 \to k^2 + 2a^2 - a^2 e^{(x-d)/\lambda}\left[1 - e^{-L/\lambda}\right]$$

$$k^2 \to k^2 + 2a^2 + a^2 e^{(d-x)/\lambda}\left[e^{L/\lambda} - 1\right]$$



$$k_0{}^2 \to k_0{}^2 + a^2\left(e^{-x/\lambda} + e^{(x-L)/\lambda}\right)$$

$$a^2 = 2\pi G \alpha_G m_n \rho \lambda^2$$



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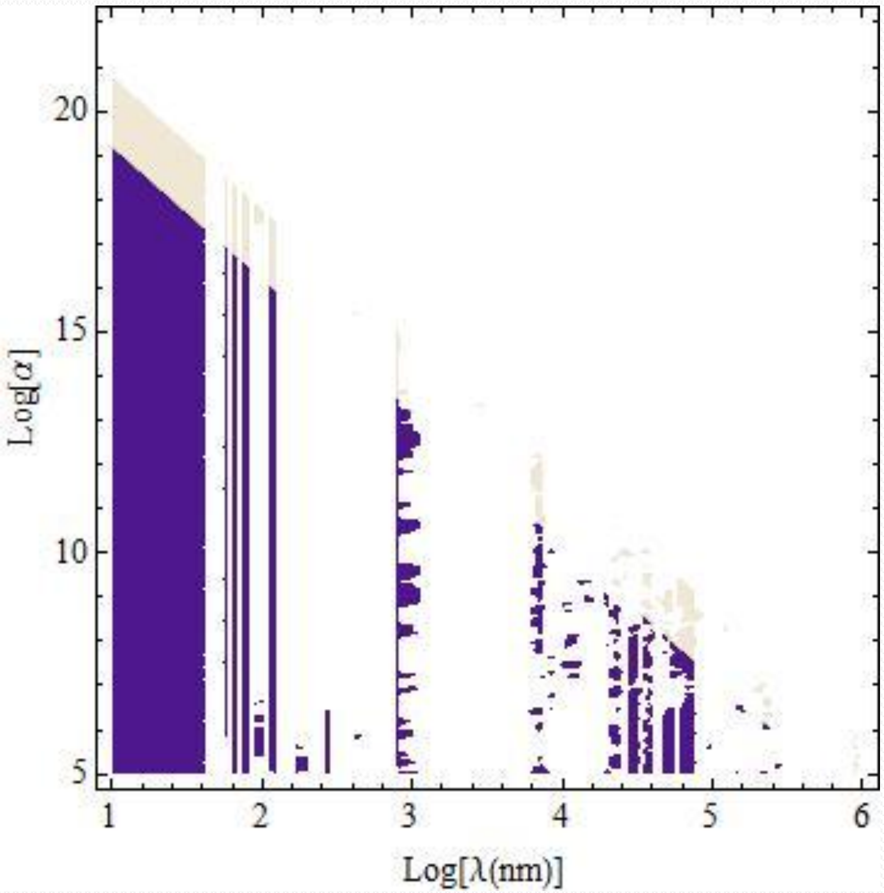
Exclusion plot for $\lambda_n = 300\,\mathring{A}$ with the experimental sensitivity to the phase at the level 10^{-3} radian by scanning the distance between the slabs from L= 0.5 λ to L= 5 λ with a step 0.5 λ



V. Gudkov, H. M. Shimizu and G. L. Greene, NIM ***A611****, 153 (2009).*

One-Dim transmission

- that transmission amplitude for exponentially decreasing potentials have infinite number of singularities in the complex momentum plane

$$k = -2i / \lambda$$

- For two overlapping potentials the transmission amplitude has a second order pole at the same position, defined by the slope.

Then, the region of maximal sensitivity is: $\lambda_n \leq \lambda$

L. D. Faddeev, Trudy Mat. Inst. Steklov **73**, 314 (1964)

M. S. Marinov & . Segev, J. Phys. A: Math. Gen. **29**, 2839 (1996)

$$T \sim t_s t_w \quad \Rightarrow \quad |T|^2 \simeq |t_s|^2 \quad \text{since } |t_w|^2 \simeq 1$$

$$Arg(T) = Arg(t_s) + Arg(t_w) + \ldots$$

$$t_w \sim 1/(k + 2i/\lambda) \quad \Rightarrow$$

$$Arg(t_w) \sim \tan^{-1}(\pi\lambda_n/\lambda)$$

$$t_w \sim 1/(k + 2i/\lambda)^2 \quad \Rightarrow$$

$$Arg(t_w) \sim \tan^{-1}\left(\frac{2(\pi\lambda_n/\lambda)}{(\pi\lambda_n/\lambda)^2 - 1}\right)$$

Quantum parametric resonance

$$d^2\psi(x)/dx^2 + k^2(x)\psi(x) = 0$$

$$k^2(x) = k_0^2(1 + 2\varepsilon \sin((2+\delta)k_0 x)) \quad \text{at} \quad |\varepsilon|, |\delta| \ll 1$$

- L. P. Pitaevsky, A. M. Perelomov, Ya. B. Zeldovich
- (L. D. Landau, E. M. Lifshitz)

Quantum PR (phase)

- The potential in between the slabs: $k_0{}^2(1+\eta\cosh(x/\lambda))$

where $\eta = 2a^2\exp(-L/2\lambda)/k_0^2$

Since $\cosh(x/\lambda) = \frac{\sinh(L/\lambda)}{(L/\lambda)} + \sum_{n=1}^{\infty}\left(\frac{2(-1)^n (L/\lambda)\sinh(L/\lambda)}{(L/\lambda)^2 + n^2\pi^2}\cos\left(\frac{n\pi x}{L}\right)\right)$

QPR conditions: $2k_0 / \left(1+\eta\frac{\sinh(L/\lambda)}{(L/\lambda)}\right) = \frac{n\pi}{L}$

or for very small parameter η: $\lambda_n \simeq 4L/n$

With the width $\gamma \simeq \frac{a^2\lambda_n^2}{\pi^2}\frac{(L/\lambda)\sinh(L/\lambda)}{(L/\lambda)^2 + 16\pi^2(L/\lambda_n)^2}e^{-L/(2\lambda)}$

“Rate” of the resonance development

$L/\lambda << 1 \quad \Rightarrow$

$$r \sim (\lambda^2 \alpha_G)^2 \lambda_n{}^4$$

$L/\lambda > 1 \quad \Rightarrow$

$$r \sim (\lambda^2 \alpha_G)^2 \lambda_n{}^4 \times \frac{e^{L/\lambda}}{L/\lambda}$$

Neutron propagation in two slabs setup

$$d^2\psi(x)/dx^2 + k^2(x)\psi(x) = 0$$

$$k^2(x) = k_0{}^2(1+\eta\cosh(x/\lambda))$$

$$k^2(x) = k_m{}^2(1+\eta_{L,R}\exp(\pm x/\lambda))$$

$$\eta_L = a^2(1-\exp(-L/\lambda))/k_m^2 \qquad \eta_R = a^2(\exp(L/\lambda)-1)/k_m^2$$

$$d^2\psi(z)/dz^2 + k^2(1+\varepsilon\cosh(z))\psi(z) = 0$$

$$d^2\psi(z)/dz^2 + k^2(1+\varepsilon\exp(z))\psi(z) = 0$$

where $z = x/\lambda$ and $k = k_0\lambda$ or $k = k_m\lambda$

$\varepsilon = \eta, \eta_R, \eta_L$

for $\varepsilon = 0$

$$\psi_0(z) = a\exp(\pm ikz) = a\exp(\pm i\varphi(z))$$

where $\varphi(z) = kz = k_0 x$

If ε>0, using Liouville's transformation*

$$\psi(z) \sim (\varepsilon k^2 \cosh(z))^{-1/4} \exp\left(\pm ik \int_{z_0}^{z} \sqrt{1+\varepsilon \cosh(z)}dz \right)(1+o(1))$$

as $z \to \infty$

then $\varphi(z) \to kz + k \sinh(z)\varepsilon / 2$

or

$$\varphi(L) \approx k_0 L + (k_0 \lambda)\varepsilon \exp(L/\lambda)/2$$

$$\varepsilon k \sim 1/k \quad \Rightarrow \quad \lambda_n \exp(L/\lambda)$$

- *(*) F. A. Berezin and M. A. Shubin, The Schrodinger Equation (Kluwer Academic Publishers, Netherlands, 1991).*

Van der Pol variables:

$$\psi = w\cos\varphi$$

$$\psi' = -kw\sin\varphi$$

$$\varphi'(z) = k + \varepsilon\cosh z\cos^2\varphi(z)$$

or $\varphi'(z) = k + \varepsilon e^z\cos^2\varphi(z)$

Non-linear equation $\Rightarrow$ instability

$$\varphi = \varphi_0 + \varepsilon\varphi_1 + \varepsilon^2\varphi_2 \ldots \quad \Rightarrow \quad \varphi_0 = kz;$$

$$\varphi_1 \sim \lambda_n^2\varepsilon'\exp(z)/2; \quad \varphi_2 \sim (\lambda_n^2\varepsilon'\exp(z))^2/4$$

Heun Equation

$$d^2\psi(x)/dx^2 + k_0{}^2(1+\eta\cosh(2k_0x))\psi(x) = 0$$

with $z = \exp(k_0 x)$ and $\psi(z) = y(z)/\sqrt{z}$

$$\Rightarrow \quad \frac{d^2y(z)}{dz^2} + \left(\frac{\eta}{2z^4} + \frac{5}{4z^2} + \frac{\eta}{2}\right)y(z) = 0$$

DCHE:

$$\frac{d^2y(z)}{dz^2} + \left(A + \frac{B}{z} + \frac{C}{z^2} + \frac{D}{z^3} + \frac{E}{z^4}\right)y(z) = 0$$

Potential of Cho and Ho[1]

$$V(x) = -\frac{b^2}{4}e^{2x} - (l+1)de^{-x} + \frac{d^2}{4}e^{-2x}$$

has an explicit phase factor[2]:

$$\sim \exp(ibe^x / 2)$$

- *1. H. T. Cho and C. L. Ho, J. Phys. A* ***40****, 1325 (2007).*
- *2. Lea Jaccoud El-Jaick, Bartolomeu D. B. Figueiredo, preprint CBPF-NF-003/08.*

The inversed power potential

$$V(x) \sim \lambda \left(\frac{x_0}{x} \right)^n$$

$$n > 2$$

$$\varphi(x) \sim \frac{2}{n-2} \left(\lambda \left(\frac{x_0}{x} \right)^{n-2} \right)^{1/2}$$

H. J. M. Muller-Kirsten, *Introduction to Quantum Mechanics. Schrodinger equation and Path Integral* (World Scientific Publishing, Singapore, 2006).
K. M. Case, Phys. Rev. ***80****, 797 (1950).*

Hill's equation

- The potential in between the slabs: $k_0{}^2(1+\eta\cosh(x/\lambda))$

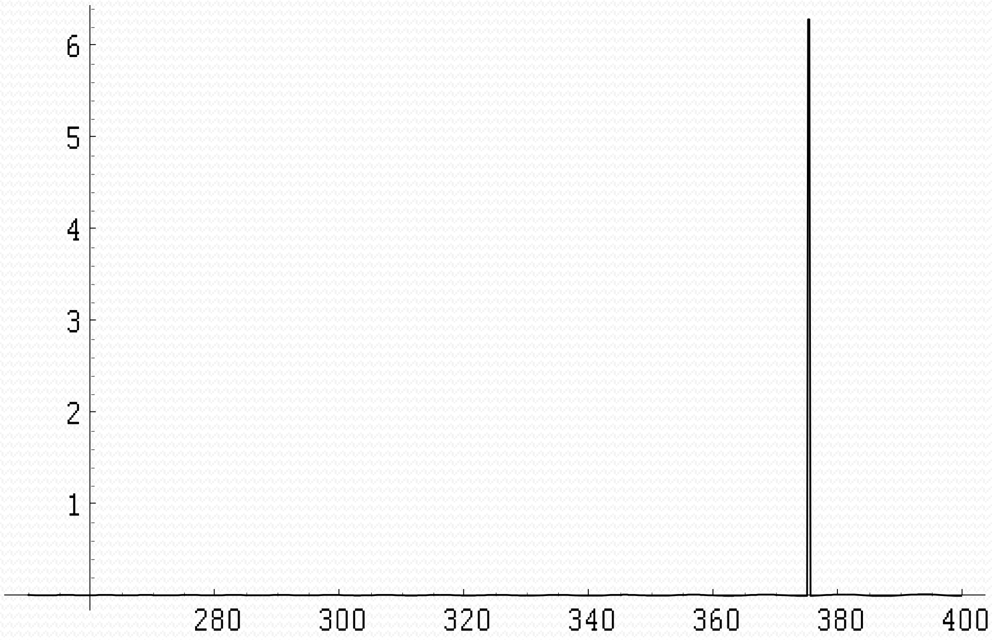
  where $\eta = 2a^2\exp(-L/2\lambda)/k_0^2$

Since $$\cosh(x/\lambda) = \frac{\sinh(L/\lambda)}{(L/\lambda)} + \sum_{n=1}^{\infty}\left(\frac{2(-1)^n(L/\lambda)\sinh(L/\lambda)}{(L/\lambda)^2+n^2\pi^2}\cos\left(\frac{n\pi x}{L}\right)\right)$$

Then:

$$\frac{d^2y(z)}{dz^2} + \left(a + \sum_{n=1}^{\infty} a_n cos(2nz)\right)y(z) = 0$$

Phase shift for two slabs with L=10nm and λ=5nm as a function of neutron wavelength λ_n ($\mathring{A}$)



V. G., H. M. Shimizu & G. L. Greene, arXiv:0709.3266

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Plot [GravDiff  [100 , 19.3 , 197 , 8, 0.01 , 1, δ, 500 , 25 ], {δ, 1, 400 }, PlotPoints   → 1000 ]
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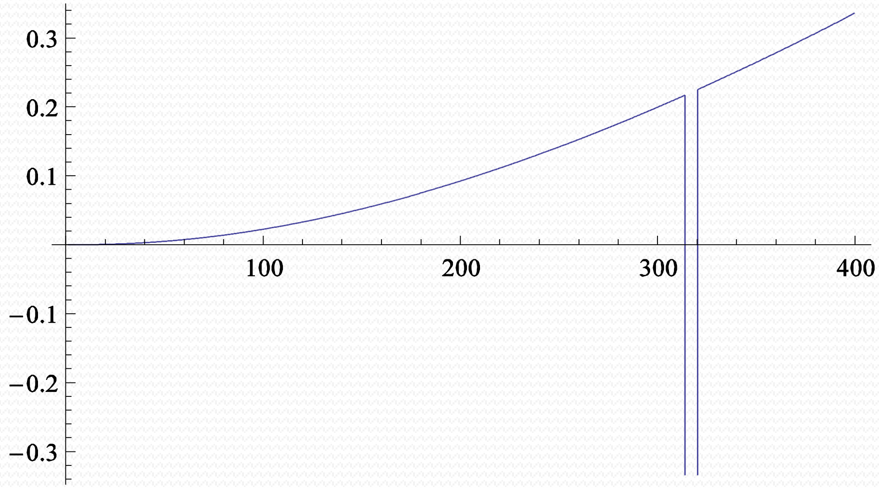

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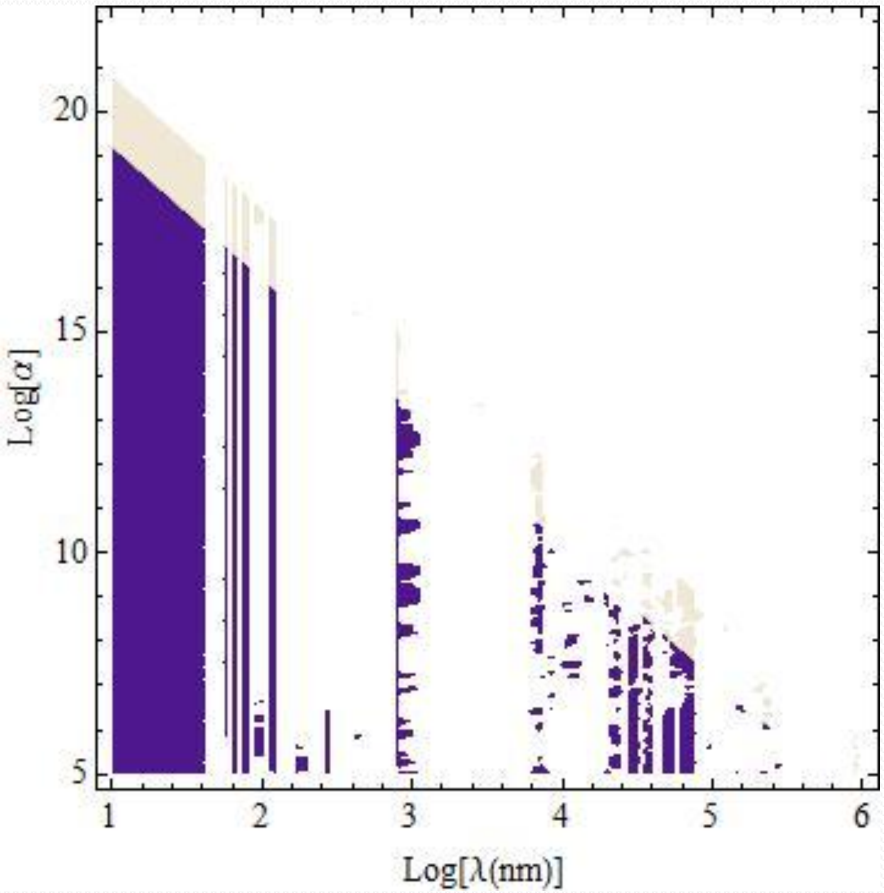

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Conclusions

UCNs are very sensitive to very week and long range interactions :

- Coherence
- Resonance – energy match
- Unstable solutions of Schrödinger's equation for neutron propagation (parametric phase enhancement) :

$$\sim \lambda_n \exp(L/\lambda) \quad \text{or} \quad \lambda_n \left(\frac{x_0}{x}\right)^{(n-2)/2}$$

$$n > 2$$